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A Generalized Solution of Asset Value Function for Capital Market Prices

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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ABSTRACT

The benefits of asset values and it return rates are geared towards investment funds which accrues wealth at any particular time and seasons. This paper studied empirically the dynamics of asset value function only on periodic events. The thorough methods which govern price function of return rate for capital funds are found .The analytical solution were obtained and verified using initial stock prices which shows: increase in volatility decreases the value of asset, a little increase in time dominantly increases the value of asset, linear rate of returns has the best estimates of asset returns. Finally, the results were subjected to goodness of fit test to show that the propose models obeys some physical laws for the purpose of investment plans.

Keywords: Asset value; kolmogorov–smirnov (KS); fourier series; stochastic analysis; prices.

1. INTRODUCTION

One means by which a commercial trader or an investor increases rate of returns is through assets. Hence, asset evaluation is an approach of determining the valuation of capital prices on a business. Asset price has become so potent for striking economic differential, allotting economic resources to sectors with time and regulating the assets of the entire system that yields different profits. In the capital market, return on investment is an evaluation which looks at the returns of a business for effective running and good management of the investment.

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However, Fourier series studies the processes where general periodic functions are represented in a form of sum of trigonometric functions. It uses the fact that any finite and time-ordered series can be completely approximated. In Fourier series analysis comparisons are made based on amplitudes and it also possible to find significant cycles with dominating amplitudes and their periods. It has so many scientific applications, for instance, mathematics in Partial differential equations, Engineering and Physics for signal processing, imaging, acoustic optics and also in finance for option pricing and some related stock variables, details of this can be found as follows: [1-3] etc.

Contrarily, while looking at this kind of problems, analytical method which offers precise solution for suitable mathematical expectation is therefore needed for the enhancement of effectiveness with regards to decision making for perspective plans. Problems concerning capital price and its rate of return, suitable formulation and correct analytical solutions are absolutely essential for the examination of return rates in time varying business; however, following the basic feature of the problem under-study, the analytical solution is exploited.

Not with standing, asset market prices have been studied in various ways and results obtained in divers1 ways by researchers. Thus, [4] examined the stochastic analysis of asset market price model, and looked at the stochastic analysis of the behavior of stock prices making use of a suggested log-normal distribution model. Their outcomes revealed that the suggested model is effective for the generation of stock prices.[5] considered the stochastic analysis of stock market predicted returns for investors. The variances of four different stocks in the results, showed that stock1 is the best among the stocks of different companies, which is consistent with the work of [6]. Also, [7] investigated a stochastic analysis of stock market expected returns and growth-rates.

In a bid to study stochastic model, [8] looked at the stability analysis of stochastic model for stock market prices and did analysis of the unstable nature of stock market forces using modern differential equation model that can impact the predicted returns of investors in stock exchange market with a stochastic volatility in the equation. Although [9] proposed in their study, some analytical solutions of stochastic differential equations concerning Martingale processes and found out that the solutions of some SDEs are related to other stochastic equations with diffusion part. The second technique is to transform SDE to ODE that tried to neglect diffusion part of stochastic equation by applying Martingale processes.

On the other hand, [10] looked at the numerical techniques of solving stochastic differential equations like the Euler-Maruyama and Milstein methods based on the truncated Ito-Taylor expansion by solving a non-linear stochastic differential equation and approximated numerical solution using Monte Carlo simulation for each scheme. Their results showed that if the discretization value N is increasing, the Euler-Maruyama and Milstein techniques were closed to exact solution [11] studied the stability of both analytical and numerical solutions for non-linear stochastic delay differential equations with jumps and they noticed that the compensated stochastic methods inherit stability property of the correct solution [12] considered the solution of differential equations and stochastic differential equations of time changing investment returns and got exact conditions controlling capital price returns rate through multiplicative and multiplicative inverse trend series. The suggested model revealed a adequate and steady multiplicative inverse trend series than the multiplicative trend in both deterministic and stochastic system [13] examined the stochastic model of the fluctuations of stock market price and obtained precise conditions for determining the equilibrium price. The model inhibits the drift parameters of price process in a manner that is adequately characterized by the volatility [14] studied the stability behaviours of stochastic differential equations (SDEs) propelled by time changing Brownian motions. And a connection between the stability of the solution to the time -changed stochastic differential equations and their corresponding non-time-changed stochastic differential equations were shown using the duality theorem [15] studied stochastic methods in applicative representation of financial models and suggested the Euler-Maruyama method as the stochastic differential equation formulations as clearly facilitative for the illustration of stock price and volatility. To a greater extent, [16,17] considered stock price forecasting and applied the method of Fourier series analysis. The result showed the level trading does not provide a practical value in comparison to the momentum trading method. The impact of Fourier series expansion on the solution of stochastic differential equation was considered by Loko et al. [18]. In another dimension [19] examined some applications of one and two-dimensional Fourier series transforms.

This paper aimed at studying empirical analysis of stock returns on periodic events for capital investments which was not considered by the previous efforts.

This paper is arranged as follows: Section 2.1 presents the preliminaries, definitions and formulations, Results and discussion are seen in Section 3.1 and paper is concluded in Section 4.1.

2. PRELIMINARIES

Definition 1: A Stochastic Differential Equation (SDE) is integration of differential equation with stochastic terms. So, in considering the Geometric Brownian Motion (GBM) which govern price dynamics of a non-dividend paying stock as:

$$dS(t) = \mu S(t)dt + \sigma S(t)dz(t) \ , \quad (1)$$

Where S denotes the asset value, μ is the stock rate of return (drift) which is also known as the average rate of the growth of asset price and σ denotes the volatility otherwise called standard deviation of the returns. The $dz(t)$ is a Brownian motion or Wiener process which is defined on probability space $(\Omega, \mathrm{F}, \wp)$, [20]. However, stock price follows the Ito's process and the drift rate is stated as follows:

$$\mu = \left(\frac{\partial f}{\partial S_t} a_t + \frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial S_T^2} b_t^2 \right), \quad (2)$$

$$\sigma^2 = \frac{\partial^2 f}{\partial S_t^2} b_t^2 \quad (3)$$

Definition 2: Ito's process is a stochastic process $\{X_t, t \geq 0\}$ known as Ito's process which follows:

$$X_t = X_0 + \int_0^t (\mathrm{t}, \varpi) d\tau + \int_0^t b(t, \varpi) dz_t \ . \quad (4)$$

Where $a(t,\varpi)$ and $b(t,\varpi)$ are adapted random function, [18]

Definition 3: (Ito's lemma). Let $f(S,t)$ be a twice continuous differential function on $[0,\infty) \times \mathrm{A}$ and let S_t denotes an Ito's process

$$dS_t = a_t dt + b_t dz(t), t \geq 0 \ ,$$

Applying Taylor series expansion of F gives:

$$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} (dS_t)^2 + \text{higer order terms } (h.\mathrm{o}, \mathrm{t}) \ ,$$

So, ignoring h.o.t and substituting for dS_t we obtain

$$dF_t = \frac{\partial F}{\partial S_t} (a_t dt + bdz(t)) + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} (a_t dt + bdz(t))^2 \quad (5)$$

$$= \frac{\partial F}{\partial S_t} (a_t dt + bdz(t)) + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} b_t^2 dt, \quad (6)$$

$$= \left(\frac{\partial F}{\partial S_t} a_t + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} b_t^2 \right) dt + \frac{\partial F}{\partial S_t} b_t dz(t) \quad (7)$$

More so, given the variable $S(\mathrm{t})$ denotes stock price, then following GBM implies (3) and hence, the function $F(S,t)$,Ito's lemma gives:

$$dF = \left(\mu S \frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 F}{\partial S^2} \right) dt + \sigma S \frac{\partial F}{\partial S} dz(t) \tag{8}$$

To determine a stock price S which follows a random process as

$$dS(t) = \mu S(t)dt + \sigma S(t)dz(t)$$

Let $F(S,t) = \ln S$ partial derivatives are:

$$\frac{\partial F}{\partial S} = \frac{1}{S}, \frac{\partial^2 F}{\partial S^2} = -\frac{1}{S^2} \quad and \frac{\partial F}{\partial t} = 0.$$

Putting the above values into (8) yields the following

$$d(\ln S) = \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma dz(t),$$

Integrating both sides and talking upper and lower bounds as 0 to t gives

$$\int_0^t d(\ln S) = \int_0^t \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma \int_0^t dz(t),$$

$$\ln S(t) - l\,\mathrm{n}\, S_0 = \left(\mu - \frac{\sigma^2}{2} \right) t + \sigma\left(dz(\mathrm{t}) - dz(0) \right),$$

$$S(t) = S_0 \exp\left(\mu - \frac{\sigma^2}{2} \right) t + \sigma dz(t). \tag{9}$$

where dz is a standard Brownian Motion.

2.1 Fourier Series Analysis on Stock Rate of Returns

Let the function of an asset price be represented as $f(S)$ which is defined on the bounded interval $S \in [-1,1]$ and outside of this interval we have $f(S+2T) = f(S)$. That is to say $f(S)$ has a periodic influences on asset price which is $2T$. Therefore, the Fourier series expansion of $f(S)$ is given as follows.

$$\frac{a_0}{2} + \sum_{n=0}^{\infty} \left[a_n Cos\left(\left(\frac{n\pi S}{T} \right) + b_n Sin\left(\frac{n\pi S}{T} \right) \right) \right] \tag{10}$$

where a_n and b_n are the Fourier coefficients given as

$$a_n = \frac{1}{T} \int_{-T}^{7} f(S) Cos \frac{n\pi S}{T} dS \quad n = 1, 2... \tag{11}$$

$$a_n = \frac{1}{T}\int_C^{C+27} f(S)\,Cos\frac{n\pi S}{T}dS \quad n = 0,1,2... \tag{12}$$

$$b_n = \frac{1}{T}\int_C^{C+27} f(S)\,Sin\frac{n\pi S}{T}dS \quad n = 1,2... \tag{13}$$

Where the lower limit C is equal to the interval of according to the definition of $f(S)$ and $C+2T$ remain as the upper limit of the interval; The details of this can be seen in the following books: [20-25] etc.

Using (13) gives a_0 ;that is

$$a_0 = \frac{1}{T}\int_{-7}^{T} f(S)dS \tag{14}$$

In a situation where the lower limit C relates to $-T$,we have $C=-T$;such that $C+2T=-T+2T=T.$ $C+2T=-T+2T=T.$ So we have the following coefficients to determine the stock variables or quantities.

$$a_n = \frac{1}{T}\int_{-T}^{7} f(S)\,Cos\frac{n\pi S}{T}dS \quad n = 0,1,2.. \tag{15}$$

$$b_n = \frac{1}{T}\int_{-T}^{7} f(S)\,Sin\frac{n\pi S}{T}dS \quad n = 1,2.. \tag{16}$$

2.2 Problem Formulations

We consider rate of returns on periodic events where new dividends will not have been declared and no new assets have been purchased then the stock return follows particular processes.

Case 1: we assume a solution where return rate of asset price is a linear function of price its self. That is $f(S_1) = S_1$ and follows Fourier series with periodic influences such as $0 < S_1 < 2\pi$.

Proposition 1.

The definition of Fourier series for the capital market investment equation

$$\frac{a_0}{2} + \sum_{n=0}^{\infty}\left[a_n Cos\left(\frac{n\pi S_1}{T} + b_n Sin\left(\frac{n\pi S_1}{T}\right)\right)\right] \tag{17}$$

$where\ f(S_1) = S_1$

Proof.

We derive rate of return as a linear function of price by determining the coefficients of (17) as follows:

$$a_n = \frac{1}{T}\int_C^{C+2T} f(S_1)\,Cos\frac{n\pi S_1}{T}dS_1 \quad = \frac{1}{\pi}\int_C^{2T} f(S_1)\,Cos\frac{n\pi S_1}{T}dS_1 \quad = \frac{1}{\pi}\int_C^{2T} S_1 CosnS_1 dS_1$$

Using Nedu's method of Integration by parts

$$P_n(S_1) = S_1 \ \ and \ \ f(S_1) = CosnS_1,$$

$$= \frac{1}{\pi}\left[(S_1)\int CosnS_1 dS_1 - (1)\int CosnS_1 dS_1\right]/_0^{2\pi} = \frac{1}{\pi}\left[S_1\left(\frac{1}{n}SinnS_1\right) - 1\left(-\frac{CosnS_1}{n^2}\right)\right]/_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\left(S_1\frac{SinnS_1}{n}\right) - 1\left(-\frac{CosnS_1}{n^2}\right)\right]/_0^{2\pi} = \frac{1}{\pi}\left[\frac{Cos2n\pi}{n^2} - \frac{1}{n^2}\right] = \frac{1}{n^2\pi}(1-1) = 0$$

$$if\ \ n = 0,\ a_0 = \frac{1}{\pi}\int_C^{27} f(S_1)dS_1 = \frac{1}{\pi}\int_0^{2\pi} S_1 dS = \frac{1}{\pi}\left[\frac{S_1^{\,2}}{2}\right]/_0^{2\pi} = 2\pi$$

Similarly

$$b_n = \frac{1}{T}\int_C^{C+2T} f\left(S_1\right) Sin \frac{n\pi S_1}{T} dS_1 \quad = \frac{1}{\pi}\int_C^{2T} S_1 SinnS_1 dS_1$$

Using Nedu's method of Integration by parts

$$P_n\left(S_1\right) = S_1 \;\; and \;\; f\left(S_1\right) = SinnS_1,$$

$$= \frac{1}{\pi}\left[\left(S_1\right)\int SinnS_1 dS_1 - (1)\int SinnS_1 dS_1\right]/_0^{2\pi} = \frac{1}{\pi}\left[S_1\left(-\frac{CosnS_1}{n}\right) - 1\left(-\frac{SinnS_1}{n^2}\right)\right]/_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{-2\pi Cos 2n\pi}{n}\right] = -\frac{2}{n}$$

$$\text{Hence } \; f\left(S_1\right) = \sum_{n=1}^{\infty} -\frac{2}{n} SinS_1$$

$$= \pi - 2\left[SinS_1 + \frac{1}{2}Sin2S_1 + \frac{1}{3}Sin3S_1 + \frac{1}{4}Sin4S_1 + \frac{1}{5}Sin5S_1 + \frac{1}{6}Sin6S_1 + ...\right] \quad (18)$$

This is the net gain or loss of an investment over a specified time period. Consequently we set $f\left(S_1\right) = \mu$ of (9) which offer a complete solution of SDE with the effect of Fourier series expansion

$$S_1(t) = S_0 \exp\left(\left(\pi - 2\left[\begin{matrix} SinS_1 + \frac{1}{2}Sin2S_1 + \frac{1}{3}Sin3S_1 + \frac{1}{4}Sin4S_1 + \frac{1}{5}Sin5S_1 \\ + \frac{1}{6}Sin6S_1 + ... \end{matrix}\right] - \frac{\sigma^2}{2}\right)t + \sigma dz(t).\right). \quad (19)$$

Case 2: we derive a solution where return rate of asset price is a quadratic function . That is $f\left(S_2\right) = {S_2}^2$ and follows Fourier series with periodic influences such as $0 < S_2 < 2\pi$.

Proposition 2.

Here let the definition of Fourier series for the capital market investment equation be given as follows:

$$\frac{a_0}{2} + \sum_{n=0}^{\infty} \left[a_n Cos\left(\frac{n\pi S_2}{T} + b_n Sin\left(\frac{n\pi S_2}{T}\right)\right)\right]. \quad (20)$$

$$where \;\; f\left(S_2\right) = {S_2}^2$$

Proof.

We want to show rate of return as a quadratic function of price by determining the coefficients of (20) as follows

$$a_n = \frac{1}{T}\int_C^{C+2T} f(S_2)Cos\frac{n\pi S_2}{T}dS_2 \quad = \frac{1}{\pi}\int_C^{2T} f(S_2)Cos\frac{n\pi S_2}{T}dS_2 \quad = \frac{1}{\pi}\int_C^{2T} S_2{}^2 CosnS_2 dS_2$$

Using Nedu's method of Integration by parts

$P_n(S_2) = S_2{}^2$ and $f(S_2) = CosnS_2$,

$$= \frac{1}{\pi}\left[\left(S_2{}^2\right)\int CosnS_2 dS_2 - (2S_2)\int CosnS_2 dS_2 + (2)\int CosnS_2 dS_2 \right] /_0^{2\pi}$$

$$= \frac{1}{\pi}\left[S_2{}^2\left(\frac{1}{n}SinnS_2\right) - 2S_2\left(-\frac{CosnS_2}{n^2} + 2\left(\frac{SinnS_2}{n^3}\right)\right)\right] /_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\left(S_2{}^2\frac{SinnS_2}{n}\right) + 2S_2\left(\frac{CosnS_2}{n^2} - \frac{2SinnS_2}{n^3}\right)\right] /_0^{2\pi} = \frac{1}{\pi}\left[\frac{2S_2Cos2nS_2}{n^2}\right] /_0^{2\pi} = \frac{1}{\pi}\left[\frac{2S_2CosnS_2}{n^2}\right] /_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{4\pi 2\pi n}{n^2}\right] = \frac{4\pi(-1)^{2n}}{n^2\pi} = \frac{4}{n^2}, n \neq 0$$

$$if \ n = 0, \ a_0 == \frac{1}{\pi}\int_0^{2\pi} S_2{}^2 dS_2 = \frac{1}{\pi}\left[\frac{S_2{}^3}{3}\right] /_0^{2\pi} = \frac{1}{n}\left(\frac{8\pi^3}{3\pi}\right) = \frac{8\pi^2}{3}$$

Similarly

$$b_n = \frac{1}{T}\int_C^{C+2T} f(S_2)Sin\frac{n\pi S_2}{T}dS_2 \quad = \frac{1}{\pi}\int_C^{2T} S_2{}^2 SinnS_2 dS_2$$

Using Nedu's method of Integration by parts

$P_n(S_2) = S_2{}^2$ and $f(S_2) = SinnS_2$,

$$= \frac{1}{\pi}\left[\left(S_2{}^2\right)\int SinnS_2 dS_2 - (2S_2)\int SinnS_2 dS_2 + (2)\int SinnS_2 dS_2 \right] /_0^{2\pi}$$

$$= \frac{1}{\pi}\left[S_2{}^2\left(-\frac{CosnS_2}{n}\right) - 2S_2\left(-\frac{SinnS_2}{n^2}\right) + 2\left(\frac{CosnS_2}{n^3}\right)\right] /_0^{2\pi}$$

$$= \frac{1}{\pi}\left[-S_2{}^2\frac{CosnS_2}{n} + 2S_2\frac{SinnS_2}{n^2} + 2\frac{CosnS_2}{n^3}\right] /_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{-4\pi^2 Cos2n\pi}{n} + \frac{2Cos2n\pi}{n^3}\right] = -\left\{\frac{4\pi}{n} + \frac{2}{n^3}\right\}Cos2\pi n = -\left\{\frac{4n^2\pi + 2}{n^3}\right\}$$

$$\text{Hence} \quad f(S_2) = \frac{8\pi^2}{3} + \sum_{n=1}^{\infty}\left[\frac{4}{n^2}CosnS_2 - \left(\frac{4n^2\pi + 2}{n^3}\right)SinnS_2\right] = \frac{8\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2}$$

$$= \frac{8\pi^2}{3} + 4CosS_2 + Cos2S_2 + \frac{4}{9}Cos3S_2 + \frac{1}{2}Cos4S_2 + \frac{4}{25}Cos5S_2 + \frac{1}{9}Cos6S_2 + ... \qquad (21)$$

Since, we are looking at rate of return that follows Fourier series with quadratic function of price. We therefore set $f(S_2) = \mu$ of (9) which results to a complete solution of SDE with the influence of Fourier series as follows.

$$S_2(t) = S_0 \exp\left(\left(\begin{array}{l}\frac{8\pi^2}{3} + 4CosS_2 + Cos2S_2 + \frac{4}{9}Cos3S_2 + \frac{1}{2}Cos4S_2 + \frac{4}{25}Cos5S_2 \\ + \frac{1}{9}Cos6S_2 + ... - \frac{\sigma^2}{2}\end{array}\right)t + \sigma dz(t).\right). \qquad (22)$$

3. RESULTS AND DISCUSSION

This Section presents the mathematical propositions 1 and 2 respectively whose solutions are in (19)-(22), Hence the following parameter values were used :

Table 1. A linear stock returns for assessing the value of assets with the following parameters through the solution below: $S_0 = 12.48,\ \mathrm{S}_1 = 20.21,\ \prod = 3.1429,\ dz = 1,\ \ t = 1, 2, 3.....,12$

$$S_1(t) = S_0 \exp\left(\left(\pi - 2\begin{bmatrix} SinS_1 + \frac{1}{2}Sin2S_1 + \frac{1}{3}Sin3S_1 + \frac{1}{4}Sin4S_1 + \frac{1}{5}Sin5S_1 \\ + \frac{1}{6}Sin6S_1 + ... \end{bmatrix} - \frac{\sigma^2}{2}\right)t + \sigma dz(t).\right).$$

Months(t)	$Vol.(\sigma)$	$S_1(t)$	$Vol.(\sigma)$	$S_1(t)$	$Vol.(\sigma)$	$S_1(t)$
1	0.5	145.2847	1.5	132.8009	2.00	121.8809
2	0.5	278.0819	1.5	253.1219	2.00	231.2819
3	0.5	410.8815	1.5	373.4415	2.00	340.6815
4	0.5	543.6825	1.5	493.7625	2.00	450.0825
5	0.5	676.4834	1.5	614.0834	2.00	559.4821
6	0.5	809.2843	1.5	734.4043	2.00	668.8843
7	0.5	942.1140	1.5	854.724	2.00	778.284
8	0.5	1074.8849	1.5	975.0449	2.00	887.6849
9	0.5	1207.6859	1.5	1095.3656	2.00	997.0859
10	0.5	1340.4868	1.5	1215.6868	2.00	1106.4868
11	0.5	1473.2865	1.5	1336.0065	2.00	1215.8865
12	0.5	1606.08741	1.5	1456.3274	2.00	1325.2874

Table 2. A quadratic stock returns for assessing the value of assets with the following parameters through the solution below:

$S_0 = 20.82,\ \mathrm{S}_2 = 30.20,\ \prod = 3.1429,\ dz = 1,\ \ t = 1, 2, 3.....,12$

$$S_2(t) = S_0 \exp\left(\begin{pmatrix} \frac{8\pi^2}{3} + 4CosS_2 + Cos2S_2 + \frac{4}{9}Cos3S_2 + \frac{1}{2}Cos4S_2 + \frac{4}{25}Cos5S_2 \\ + \frac{1}{9}Cos6S_2 + ... - \frac{\sigma^2}{2} \end{pmatrix}t + \sigma dz(t).\right).$$

Months(t)	$Vol.(\sigma)$	$S_2(t)$	$Vol.(\sigma)$	$S_2(t)$	$Vol.(\sigma)$	$S_2(t)$
1	0.5	3807.7073	1.5	3786.8665	2.00	3768.6490
2	0.5	7594.5530	1.5	7552.9130	2.00	7516.4780
3	0.5	11381.4196	1.5	11318.9596	2.00	11264.3071
4	0.5	15168.2861	1.5	15085.0061	2.00	15012.1361
5	0.5	18955.1526	1.5	18851.0526	2.00	18759.9651
6	0.5	22742.0191	1.5	22617.0991	2.00	22507.7941
7	0.5	26528.8856	1.5	26383.1456	2.00	26255.6231
8	0.5	30276.4856	1.5	30149.1922	2.00	30003.4522
9	0.5	34102.6187	1.5	33915.2387	2.00	33751.2812
10	0.5	37889.4852	1.5	37681.2852	2.00	37499.1102
11	0.5	41676.3517	1.5	41447.3317	2.00	41246.9392
12	0.5	45463.2182	1.5	45213.3782	2.00	44994.7682

Clearly Tables 1 and 2 describes financial markets activities in respect to various investment patterns and changes. The two tables show that increase in volatility reduces the value of assets. This is paramount, because the significant changes of stock volatility causes panic buying and fear of investing. Investors may not invest huge sum of money and some may not even invest at all. This remarks causes reduction of asset value hence it reduces drastically which a negative impact to an investment which is indexed in millions of naira or its equivalence. On the other hand, a little increase in time of asset also increases its value [26,27]. This is quite realistic because as time continue to grow most financial assets like lands, houses, agricultural products, oil etc continue to appreciate in value.

However, Table 1 has the best assets value estimates hence the numerical results are less than Table 2, statistically Table 1 will be more reliable in terms of decision making. Thereby informing investors on better investment plans when rate of returns follow linear or quadratic function series; this accounts mainly for periodic events in time varying investments. In all, the dynamic nature of asset price changes are due to stochastic formation which revolves round period returns.

H_0 : The linear $S_1(t)$ and quadratic $S_2(t)$ asset value functions comes from a common distribution

H_1 : They are not from a common distribution

Table 3. Goodness of fit test for two asset values according to trading days and volatility changes

t	$Vol.(\sigma)$	$S_1(t)$	$S_2(t)$	**Mean**	**STD**	**Kstat**	**P-Value**	**Decision**
1	0.5	145.2847	3807.7073	0.00197	0.00259	0.2890	1.0000	Accept
	1.5	132.8009	3786.8665	0.00960	0.00258	0.2890	1.0000	Accept
	2.0	121.8809	3768.6490	0.001945	0.00257	0.2890	1.0000	Accept
2	0.5	278.0819	7594.5530	0.003936	0.005174	0.2890	1.0000	Accept
	1.5	253.1219	755.9130	504.5174	355.5270	0.2890	1.0000	Accept
	2.0	231.2819	7516.4780	0.003874	0.005151	0.2890	1.0000	Accept
3	0.5	410.8815	11381.4196	0.005896	0.007757	0.2890	1.0000	Accept
	1.5	373.4415	11318.9596	0.005846	0.007740	0.2890	1.0000	Accept
	2.0	340.6815	11264.3071	0.005803	0.007724	0.2890	1.0000	Accept
4	0.5	543.6825	15168.2861	0.007856	0.0001034	0.2890	1.0000	Accept
	1.5	493.7625	15085.0061	0.007894	0.0001032	0.2890	1.0000	Accept
	2.0	450.0825	15012.1361	0.005803	0.007724	0.2890	1.0000	Accept
5	0.5	676.4834	18955.1526	0.009816	0.0001293	0.2890	1.0000	Accept
	1.5	614.0834	18851.0526	0.009733	0.0001293	0.2890	1.0000	Accept
	2.0	559.4821	18759.9651	0.009660	0.0001287	0.2890	1.0000	Accept
6	0.5	809.2843	22742.0191	0.009660	0.0001551	0.2890	1.0000	Accept
	1.5	734.4043	22617.0991	0.0001776	0.0001547	0.2890	1.0000	Accept
	2.0	668.8843	22507.7941	0.0001159	0.0001544	0.2890	1.0000	Accept

The alternative hypothesis of KS was clearly accepted since its p-values are not significant. The results displayed in Table 3 show that at $\alpha = 0.01$ the two asset values do not come from a common distribution which has financial implications on the aspect of asset returns; with this guide investors will be well informed on the issues of decision making. We therefore conclude that there is significant difference between the two asset values $\left(S_1(t) \text{ and } S_2(t)\right)$.

However, the mean rates of returns seen in column 5 of Tables 1 and 2 respectively indicates the likely returns of financial investments at different maturity days; each asset was traded. This remark informs an investor or financial analyst the current value of assets at time t with variations of volatility changes.

4. CONCLUSION

The advantage of monetary assets and it return rates is geared towards investment funds which accrues wealth periodically .This paper studied empirically the dynamics of asset value function only on periodic events. The analytical solution were obtained and verified using initial stock prices which shows: increase in volatility decreases the value of asset, a little increase in time dominantly increases the value of asset, linear rate of returns has the best estimates of asset returns in respect to decision making. However, the results were subjected to certain statistical test to show that the propose models obeys some physical laws.

We shall be looking at the uniqueness of this two prepositions and incorporating delay terms in the next study.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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